

Unitary Transformation in Probabilistic Teleportation

Xiu-Lao Tian*, Wei Zhang, Xiao-Qiang Xi

School of Science, Xi'an University of Posts and Telecommunications, Xi'an 710121, China

September 11, 2012

Abstract

We proposed a general transformation in probabilistic teleportation, which is based on different entanglement matching coefficients K corresponding to different unitary evolution which provides one with more flexible evolution method experimentally. Through analysis based on the Bell basis and generalized Bell basis measurement for two probabilistic teleportation, we suggested a general probability of successful teleportation, which is not only determined by the entanglement degree of transmission channels and measurement methods, but also related to the unitary transformation in the teleportation process.

Keywords: probabilistic teleportation; entanglement matching; channel parameter matrix
PACS: 03.67.Hk, 03.65.Ta

1 Introduction

Quantum entanglement is one of the most fascinating characteristic of quantum physics, a fantastic application of entanglement [1, 2, 3] is quantum teleportation, which plays a key role in the field of quantum communication. Since the seminal work of Bennett et al. [4], teleportation has been the research interest of researchers and a number of work both in theory and experiments has been devoted to it [5, 6, 7, 8, 9, 10, 11].

Up to now the teleportation has been studied in different branches, such as directly and network controlled teleportation [12, 13, 14, 15]; discrete-variables and continuous-variables teleportation [16, 17, 18]; perfect and probabilistic teleportation [19, 20] and so on. In fact, one of the key problem of teleportation is how to construct an usefulness quantum channel, different channels will yield different results, some channels can be used to realize perfect teleportation, while some others can only enable probabilistic teleportation. Because of the inevitable interaction with its surroundings, correlations in quantum states are difficult to maintain [21, 22], therefore the probabilistic teleportation [23, 24, 25, 26, 27] has been widely discussed in recent years.

A necessary and sufficient condition for realizing perfect teleportation and successful teleportation has been given in [28, 29, 30, 31]. Based on the Bell basis measurement, we found that if the channel parameter matrix (CPM) is unitary, then one can always realize a perfect teleportation (i.e., the successful probability $p = 1$), if the CPM is invertible but not unitary, however, one can only realize a probabilistic teleportation (i.e., the successful probability $p < 1$).

*Corresponding author: E-mail, txl@xupt.edu.cn

Motivated by the idea of Ref. [19], in which the authors introduced an auxiliary qubit to realize probabilistic teleportation, we propose some unitary transformation methods in probabilistic teleportation in this work. These methods are based on different entanglement matching coefficients K which corresponding to different unitary transformations. Through detailed analysis with the Bell basis and generalized Bell basis measurement in two probabilistic teleportation processes, we suggest a general probability of successful teleportation, which is not only determined by the entanglement degree of transmission channel and measurement methods, but also related to the unitary transformation in the teleportation process. For example, if we teleport the unknown one-qubit state $|\varphi\rangle_1$ via the channel state $|\varphi\rangle_{2,3} = a|00\rangle + b|11\rangle$, then the whole probability of successful teleportation is $P = 2(Kab)^2$, where $0 < K \leq \min(\frac{1}{|a|}, \frac{1}{|b|})$. Our conclusion covers and complements the results of Ref. [19]. As different K will give different kinds of evolution methods, one can have more flexible and selectable evolution method experimentally.

2 Entanglement matching and probabilistic teleportation

Suppose Alice wants to send an unknown one-qubit state $|\varphi\rangle_1$ to Bob

$$|\varphi\rangle_1 = R^i|i\rangle = R^0|0\rangle + R^1|1\rangle = \alpha|0\rangle + \beta|1\rangle, \quad (1)$$

where $R^i R_i^* = |\alpha|^2 + |\beta|^2 = 1$, with i being taken to be 0 or 1 and a repeated index denotes summation.

The general two-qubit state $|\varphi\rangle_{2,3}$ used as the quantum channel can be expressed as follows

$$|\varphi\rangle_{2,3} = \frac{1}{\sqrt{2}} X^{jk} |jk\rangle = \frac{1}{\sqrt{2}} (X^{00}|00\rangle + X^{01}|01\rangle + X^{10}|10\rangle + X^{11}|11\rangle), \quad (2)$$

where $X^{jk} X_{jk}^* = 2$. If Alice adopts the standard Bell basis measurement (BM) ϕ_{ij}^λ ($\lambda = 1, 2, 3, 4$) on her particles, i.e., $\phi_{ij}^{1,2} = (|00\rangle \pm |11\rangle)/\sqrt{2}$ and $\phi_{ij}^{3,4} = (|01\rangle \pm |10\rangle)/\sqrt{2}$. Then with the denotation of Bell basis [28, 29, 30], the total state of the system can be rewritten as

$$|\Psi\rangle_{tot} = \frac{1}{\sqrt{2}} R^i X^{jk} L_{ij}^\lambda |\lambda k\rangle = \frac{1}{2} R^i X^{jk} T_{ij}^\alpha |\alpha k\rangle = \frac{1}{2} R^i \sigma_i^{(\lambda)k} |\alpha k\rangle, \quad (3)$$

where $\sigma_i^{(\lambda)k} = X^{jk} T_{ij}^\lambda = X^{jk} \sqrt{2} L_{ij}^\alpha$ is the element of

$$\sigma^\alpha = X T^\alpha = \begin{pmatrix} \sigma_0^{\lambda 0} & \sigma_1^{\lambda 0} \\ \sigma_0^{\lambda 1} & \sigma_1^{\lambda 1} \end{pmatrix} = \begin{pmatrix} X^{00} & X^{10} \\ X^{01} & X^{11} \end{pmatrix} \begin{pmatrix} T_{00}^\lambda & T_{10}^\lambda \\ T_{01}^\lambda & T_{11}^\lambda \end{pmatrix}. \quad (4)$$

After Alice's measurement, the total state will collapse to

$$|\Psi^\alpha\rangle_B = \frac{1}{2} R^i \sigma_i^{(\alpha)k} |k\rangle. \quad (5)$$

Obviously, based on the BM method, all the T^α are unitary. So if the CPM X is unitary, one can always realize the perfect teleportation (i.e., the whole probability $p = 1$), if X is invertible but not unitary, one can only realize a probabilistic teleportation (i.e., the whole probability $p < 1$).

In Ref. [19]. Li et al. presented a protocol of probabilistic teleportation by introducing an auxiliary qubit state $|0\rangle_A$ and performing an unitary transformation on Bob's state. They employ a partially entangled state as the quantum channel, that is

$$|\varphi\rangle_{2,3} = X^{jk} |jk\rangle = a|00\rangle + b|11\rangle \quad (a \neq b), \quad (6)$$

with a, b being real numbers and $a^2 + b^2 = 1$. The CPM $X = \sqrt{2}\text{diag}(a, b)$ is obviously invertible but not unitary, so Bob cannot directly retrieve the state by acting $(\sigma^\alpha)^{-1}$ on the collapsed state $|\Psi^\alpha\rangle_B$.

With the standard Bell basis ϕ_{ij}^λ ($\lambda = 1, 2, 3, 4$), the total state of the system can be rewritten as

$$\begin{aligned} |\Psi\rangle_{tot} &= \frac{1}{\sqrt{2}} R^i X^{jk} T_{ij}^\lambda |\lambda k\rangle = \frac{1}{\sqrt{2}} R^i \sigma_i^{(\lambda)k} |\lambda k\rangle \\ &= \frac{1}{\sqrt{2}} [\phi_{1,2}^1(a\alpha|0\rangle + b\beta|1\rangle) + \phi_{1,2}^2(a\alpha|0\rangle - b\beta|1\rangle) \\ &\quad + \phi_{1,2}^3(a\beta|0\rangle + b\alpha|1\rangle) + \phi_{1,2}^4(a\beta|0\rangle - b\alpha|1\rangle)]. \end{aligned} \quad (7)$$

After Alice's BM ϕ_{12}^1 , Bob will get the unnormalized state as follows

$$|\Psi^1\rangle_b = \frac{1}{\sqrt{2}} R^i \sigma_i^{(1)k} |k\rangle = \frac{1}{\sqrt{2}} (a\alpha|0\rangle + b\beta|1\rangle). \quad (8)$$

In order to obtain the original state, one can introduce an auxiliary qubit state $|0\rangle_A$ [19] in Bob's state, now Bob's state can be express as

$$|\Psi^1\rangle_{A,3} = \frac{1}{\sqrt{2}} R^i \sigma_i^{(1)k} |k\rangle = \frac{1}{\sqrt{2}} |0\rangle_A [(a\alpha|0\rangle + b\beta|1\rangle)_3], \quad (9)$$

With the standard two-qubit basis $(|00\rangle, |01\rangle, |10\rangle, |11\rangle)_{A,3}$, we propose an unitary transformation U with parameter K for the particles (A,3)

$$U = \begin{pmatrix} Kb & 0 & \sqrt{1-(Kb)^2} & 0 \\ 0 & Ka & 0 & \sqrt{1-(Ka)^2} \\ \sqrt{1-(Kb)^2} & 0 & -Kb & 0 \\ 0 & \sqrt{1-(Ka)^2} & 0 & -Ka \end{pmatrix}. \quad (10)$$

where we called K the entanglement matching coefficient of Bob's evolution. To ensure the transformation U to be unitary, we demand $0 < K \leq \min(\frac{1}{|a|}, \frac{1}{|b|})$. There are different unitary transformation methods with different K .

After Bob's evolution, the state $|\Psi^1\rangle_{A,3}$ turns out to be

$$|\Psi^1\rangle_{A,3} = \frac{1}{\sqrt{2}} [Kab|0\rangle_A(\alpha|0\rangle + \beta|1\rangle)_3 + a\sqrt{1-(Kb)^2}\alpha|1\rangle_A|0\rangle_3 + b\sqrt{1-(Ka)^2}\beta|1\rangle_A|1\rangle_3]. \quad (11)$$

Certainly, $|\Psi^1\rangle_{A,3}$ is not normalized. Now Bob performs measurement on the auxiliary qubit A , if the measurement outcome is $|1\rangle_A$, the teleportation fails, if the measurement outcome is $|0\rangle_A$, the teleportation is successfully accessed and Bob's state becomes

$$|\Psi^1\rangle_3 = \frac{1}{\sqrt{2}} Kab(\alpha|0\rangle + \beta|1\rangle)_3 \quad (12)$$

Now we discuss the probability of successful teleportation, which contains both Alice's P_A and Bob's P_B probability. From Eq. (7) one can obtain the Bell state $\phi_{1,2}^1$ occurring probability as

$$P_A^1 = \frac{1}{2} \langle 0|a\alpha\langle 1|b\beta(|a\alpha|0\rangle + b\beta|1\rangle)_3 = \frac{1}{2} [(a\alpha)^2 + (b\beta)^2], \quad (13)$$

Similarly, the Bell state $\phi_{1,2}^2$, $\phi_{1,2}^3$ and $\phi_{1,2}^4$ occurring probability are

$$P_A^2 = P_A^1 = \frac{1}{2}[(a\alpha)^2 + (b\beta)^2], P_A^3 = P_A^4 = \frac{1}{2}[(a\beta)^2 + (b\alpha)^2], \quad (14)$$

If $a = b = \frac{1}{\sqrt{2}}$, then $P_A^1 = P_A^2 = P_A^3 = P_A^4 = \frac{1}{4}$, which is just the prefect teleportation.

Now we compute the probability of Bob for obtaining the original state from the state $|\Psi^1\rangle_{A,3}$. The normalized state corresponding to the state in Eq. (11) is

$$|\Psi^1\rangle_{(A,3)norm} = \frac{1}{\sqrt{a^2\alpha^2 + b^2\beta^2}}|\Psi^1\rangle_{A,3}. \quad (15)$$

After Bob's successful measurement on $|0\rangle_A$, the probability P_B^1 of obtaining the original state from the state $|\Psi^1\rangle_{A,3}$ is

$$P_B^1 = \frac{(Kab)^2}{(a\alpha)^2 + (b\beta)^2}. \quad (16)$$

We consider Alice's measurement and Bob's different operations in the teleportation process. For Alice's each measurement and Bob's operation, the probability of obtaining the initial state is

$$P_{AB}^1 = P_A^1 P_B^1 = \frac{1}{2}(Kab)^2 \quad (17)$$

where P_{AB}^1 is just the square of coefficient of the state $|\Psi^1\rangle_3$ in Eq. (12).

Summing all the contributions of $P_{AB}^1 = P_{AB}^2 = P_{AB}^3 = P_{AB}^4 = \frac{1}{2}(Kab)^2$, we obtain the whole probability of successful teleportation as

$$P = \sum_{i=1}^4 = 2(Kab)^2 \quad (18)$$

where $0 < K \leq \min(\frac{1}{|a|}, \frac{1}{|b|})$.

If $a > b$, we take $K = K_{max} = \frac{1}{a}$, then the optimal probability is $P = 2b^2$. When $a = b = \frac{1}{\sqrt{2}}$, then $P_{max} = 1$, which is just the case for prefect teleportation, and for this special case U is the same as that in Ref. [19], i.e.

$$U = \begin{pmatrix} b/a & 0 & \sqrt{1 - (b/a)^2} & 0 \\ 0 & 1 & 0 & 0 \\ \sqrt{1 - (b/a)^2} & 0 & -b/a & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \quad (19)$$

For different matching coefficients K , one can adopt different kinds of unitary transformation. For example, when $K = 1$ the whole probability of successful teleportation is $P = 2(ab)^2$ and our unitary transformation matrix turns out to be

$$U = \begin{pmatrix} b & 0 & a & 0 \\ 0 & a & 0 & b \\ a & 0 & -b & 0 \\ 0 & b & 0 & -a \end{pmatrix}. \quad (20)$$

Next we discuss some difference between the probability $P = 2b^2$ and $P = 2(Kab)^2$ for two kinds of unitary transformation. For arbitrary $a^2 + b^2 = 1$ and $a \neq b$, there are always $P = 2b^2 > 2(ab)^2$. So Eq. (20) is an unitary transformation for obtaining the optimal probability. However, the condition $a^2 + b^2 = 1$ and $a > b$ yields $b^2 < 1/2$, and therefore one can only obtain probability $P < 1$. When $K = 1$ we have $P = 2(ab)^2$, and the normalization condition $a^2 + b^2 = 1$ gives rise to $P_{max} = 1/2$ with $a^2 = b^2 = 1/2$, thus one can only attain the probability $P = 2(ab)^2 < 1/2$ for $a \neq b$.

Because $0 < K \leq \min(\frac{1}{|a|}, \frac{1}{|b|})$, when $1 \leq K \leq 2$ (here $a > b$ and $a \sim b$), then $P = 4(ab)^2 \sim 2b^2$. For this case there are a little difference between the probability $P = 2b^2$ and $P = 4(ab)^2$ for the two kinds of unitary transformation (see Fig. 1).

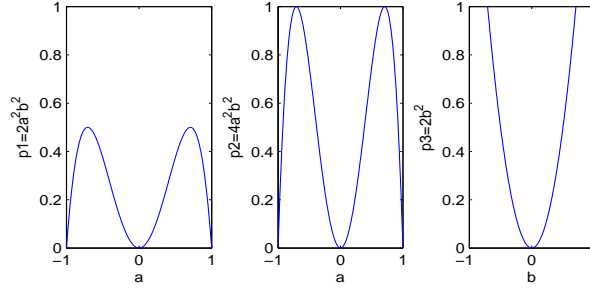


Fig. 1. Comparison of the probability with different channel parameters.

Different matching coefficients K correspond to different unitary transformations. Although $P = 2b^2 > 2(Kab)^2$, it provides one with more flexible selection for probabilistic teleportation.

3 Probabilistic teleportation with generalized BM

Considering now Alice makes a generalized Bell basis measurement (GBM), these are

$$\phi_{1,2}^1 = a'|00\rangle + b'|11\rangle, \quad \phi_{1,2}^2 = b'|00\rangle - a'|11\rangle, \quad \phi_{1,2}^3 = a'|01\rangle + b'|10\rangle, \quad \phi_{1,2}^4 = b'|01\rangle - a'|11\rangle. \quad (21)$$

where $a'^2 + b'^2 = 1$ and $a' \neq b'$. The transformation matrix T between the generalized Bell basis and computation basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ is

$$T = \begin{pmatrix} a' & 0 & 0 & b' \\ b' & 0 & 0 & -a' \\ 0 & a' & b' & 0 \\ 0 & b' & -a' & 0 \end{pmatrix}. \quad (22)$$

Let us reconsider the aforementioned one-qubit teleportation, under the generalized Bell basis, the total state of the system is

$$|\Psi\rangle_{tot} = \frac{1}{\sqrt{2}} R^i X^{jk} T_{ij}^\lambda |\lambda k\rangle = \frac{1}{2} R^i \sigma_i^{(\lambda)k} |\lambda k\rangle. \quad (23)$$

After Alice's GBM ϕ_{12}^λ , Bob will get the corresponding unnormalized state as follows

$$\begin{aligned} |\Psi^1\rangle_B &= (aa'\alpha|0\rangle + bb'\beta|1\rangle), & |\Psi^2\rangle_B &= (ab'\alpha|0\rangle - ba'\beta|1\rangle), \\ |\Psi^3\rangle_B &= (aa'\beta|0\rangle + bb'\alpha|1\rangle), & |\Psi^4\rangle_B &= (ab'\beta|0\rangle - ba'\alpha|1\rangle). \end{aligned} \quad (24)$$

When Bob introduced an auxiliary qubit state $|0\rangle_A$ for $|\Psi^\alpha\rangle_B$, and make an unitary transformation U_1 to the state $|\Psi^1\rangle_{A,3}$, with

$$U_1 = \begin{pmatrix} Kbb' & 0 & \sqrt{1-(Kbb')^2} & 0 \\ 0 & Kaa' & 0 & \sqrt{1-(Kaa')^2} \\ \sqrt{1-(Kbb')^2} & 0 & -Kbb' & 0 \\ 0 & \sqrt{1-(Kaa')^2} & 0 & -Kaa' \end{pmatrix}, \quad (25)$$

where $0 < K \leq \min(\frac{1}{|aa'|}, \frac{1}{|bb'|})$, the state $|\Psi^1\rangle_{A,3}$ becomes

$$\begin{aligned} |\Psi^1\rangle_{A,3} &= \frac{1}{\sqrt{2}}|0\rangle_A[Kaba'b'(\alpha|0\rangle + \beta|1\rangle)_3 \\ &+ a\sqrt{1-(Kbb')^2}\alpha|1\rangle_A|0\rangle_3 + b\sqrt{1-(Kaa')^2}\beta|1\rangle_A|1\rangle_3. \end{aligned} \quad (26)$$

If Bob's measurement outcome is $|0\rangle_A$, the teleportation is successfully implemented.

Considering Alice's Bell basis measurement ϕ^1 and Bob's evolution on $|\Psi^1\rangle_{A,3}$, the probability of successful teleportation is

$$P_{AB}^1 = (Kaba'b')^2. \quad (27)$$

Similarly, After Bob's U_2 , U_3 and U_4 transformation to the corresponding states and measurement on the auxiliary qubit, the probability of successful teleportation respectively are

$$P_{AB}^2 = P_{AB}^3 = P_{AB}^4 = (Kaba'b')^2. \quad (28)$$

Thus the whole probability of successful teleportation is

$$P = 4(Kaba'b')^2. \quad (29)$$

Next we discuss the optimal probability of successful teleportation by entanglement matching for the following two cases:

(1) $|a| \geq |a'| \geq |b'| \geq |b|$. For this case, $|aa'| \geq |bb'|$, $|ab'| \geq |ba'|$, so we take $K = \frac{1}{|aa'|}$ in U_1 and U_3 , $K = \frac{1}{|ab'|}$ in U_2 and U_4 , for which one can obtain $P_{AB}^1 = P_{AB}^3 = |bb'|^2$, $P_{AB}^2 = P_{AB}^4 = |ba'|^2$. The optimal whole probability is

$$P = \sum p_i = |bb'|^2 + |ba'|^2 + |bc'|^2 + |bd'|^2 = 2|b|^2. \quad (30)$$

(2) $|a'| \geq |a| \geq |b| \geq |b'|$. In this case, $|aa'| \geq |bb'|$, $|ba'| \geq |ab'|$, so we take $K_1 = \frac{1}{|aa'|}$, $K_2 = \frac{1}{|ba'|}$, for which we have $P_{AB}^1 = P_{AB}^3 = |bb'|^2$, $P_{AB}^2 = P_{AB}^4 = |ab'|^2$. The optimal whole probability is

$$P = \sum p_i = 2|b'|^2. \quad (31)$$

From the above analysis, one can see that the optimal probability of successful teleportation is determined by the smaller value of $|b|$ and $|b'|$, i.e., the optimal probability is determined by the entanglement degree of Alice's measurement or the quantum channel. However, for $K < K_{max} = \min(\frac{1}{|aa'|}, \frac{1}{|bb'|})$, the whole probability of successful teleportation is $P = 4(Kaba'b')^2$. Therefor, the general probability of successful teleportation is not only determined by the factors of the channel and measurement, but also related to the unitary transformation during teleportation process.

4 Conclusion

The unavoidable influence of environment always induces degradation of quantum correlations, therefore the study of probabilistic teleportation is significant for quantum information processing. In this paper, we generalized the protocol of probabilistic teleportation by introducing an auxiliary qubit and the unitary transformation methods. Moreover, through the analysis based on the Bell basis and generalized Bell basis measurement in two probabilistic teleportation, we suggested a general probability of successful teleportation, which is not only determined by both the entanglement degree of transmission channels and the measurement methods, but also related to unitary transformation in teleportation process, i.e., $P = 2(Kab)^2$, Although $P = 2(Kab)^2 < 2(b)^2$ (the optimal U transformation). However in experiment, it is more important to realize successful teleportation. As different entanglement matching coefficients K will give different U evolution methods, so one can have more flexible selectable evolution method experimentally.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Grant No. 10902083, the Natural Science Foundation of Shaanxi Province under Grant Nos. 2009JQ8006, 2009JM6001 and 2010JM1011.

References

- [1] R. Horodecki, P. Horodecki, M. Horodecki, K. Horodecki, Rev. Mod. Phys. 81 (2009) 865.
- [2] A. Abliz, H. J. Gao, X. C. Xie, Y. S. Wu, W. M. Liu, Phys. Rev. A 74 (2006) 052105.
- [3] Z. G. Li, S. M. Fei, Z. D. Wang, W. M. Liu, Phys. Rev. A 79 (2009) 024303.
- [4] C. H. Bennett, G. Brassard, C. Crépeau *et al.*, Phys. Rev. Lett. 70 (1993) 1895.
- [5] D. Bouwmeester, J. W. Pan, K. Mattle *et al.*, Nature 390 (1997) 575.
- [6] M. A. Nielsen, E. Knill, R. Laflamme, Nature 396 (1998) 52.
- [7] H. Y. Dai, P. X. Chen, C. Z. Li, J. Opt. B: Quantum Semiclass. Opt. 6 (2004) 106.
- [8] Y. Yeo, W. K. Chua, Phys. Rev. Lett. 96 (2006) 060502.
- [9] Z. S. Yuan, *et al.*, Nature Phys 454 (2008) 1098.
- [10] M. L. Hu, Phys. Lett. A 375 (2011) 922.
- [11] M. L. Hu, Phys. Lett. A 375 (2011) 2140.
- [12] A. Karlsson, M. Bourennane, Phys. Rev. A 58 (1998) 4394 .
- [13] F. G. Deng, *et al.*, Phys. Rev. A 72 (2005) 022338.
- [14] Z. X. Man, Y. J. Xia, N. B. An, Phys. Rev. A 75 (2007) 052306.
- [15] G. F. Shi, X. Q. Xi, X. L. Tian, R. H. Yue, Opt. Commun. 282 (2009) 2460.
- [16] S. L. Braunstein, H. J. Kimble, Phys. Rev. Lett. 80 (1998) 869.

- [17] P. Marian, T. A. Marian, Phys. Rev. A 74 (2006) 042306.
- [18] P. Marian, T. A. Marian, Int. J. Quantum Inf. 6 (2008) 721.
- [19] W. L. Li, C. F. Li, G. C. Guo, Phys. Rev. A 61 (2000) 34301.
- [20] B. S. Shi, Y. K. Jiang, G. C. Guo, Phys. Lett. A 268 (2000) 161.
- [21] M. L. Hu, Phys. Lett. A 374 (2010) 3520.
- [22] M. L. Hu, H. Fan, Ann. Phys. 327 (2012) 851.
- [23] H. Lu, Chin. Phys. Lett. 18 (2001) 1004.
- [24] J. Fang, Y. Lin, S. Zhu, X. Chen, Phys. Rev. A 67 (2003) 014305.
- [25] T. Gao, Z. X. Wang, F. L. Yan, Chin. Phys. Lett. 20 (2003) 2094.
- [26] F. L. Yan, H. W. Ding, Chin. Phys. Lett. 23, 17 (2006).
- [27] W. X. Jiang, *et al.* Chin. Phys. Lett 24 (2007) 1144.
- [28] X. L. Tian, X. Q. Xi, Int. J. Quantum Inform. 7 (2009) 927.
- [29] X. L. Tian, *et al.*, Opt. Commun. 282 (2009) 4815.
- [30] X. L. Tian, X. Q. Xi, Mod. Phys. Lett. B 23 (2009) 2261.
- [31] X. L. Tian, M. L. Hu, X. Q. Xi, Int. J. Theor. Phys. 48 (2009) 2610.